Patterns and Connections in Developmental Mathematics

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Number Sequences
and
Inductive Reasoning

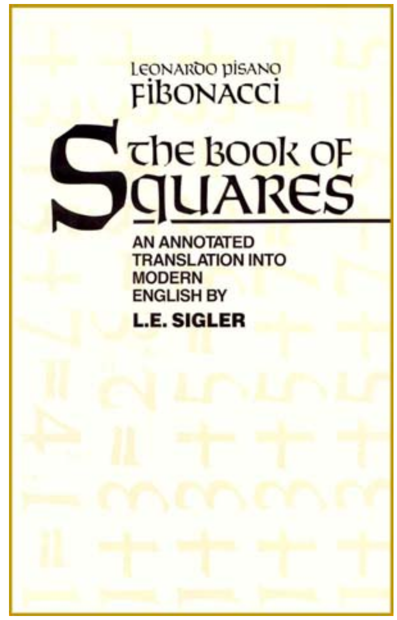
Sequence of Odd Numbers

1, 3, 5, 7, . . .

Sequence of Squares

1, 4, 9, 16, . . .

Introduction



I thought about the origin of all square numbers and discovered that they arise out of the increasing sequence of odd numbers; for the unity is a square and from it is made the first square, namely 1; to this unity is added 3, making the second square, namely 4, with root 2; if to the sum is added the third odd number, namely 5, the third square is created, namely 9, with root 3; and thus sums of consecutive odd numbers and a sequence of squares always arise together in order.

Odds: 1, 3, 5, 7, . . .

Squares: 1, 4, 9, 16, . . .

$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

Fibonacci Sequence

1, 1, 2, 3, 5, 8, . . .

Application: Count the number of bees in each generation of the family tree of a male honey bee. (A male honey bee has only one parent, a mother. A female honey bee has two parents, a mother and a father.)

§3. 곱셈공식

다항식의 곱셈을 하는 데, 앞 절의 공식이 기초가 된다. 그런데 특수한 모양의 곱셈은 따로 공식을 만들어서, 그것을 활용하면 계산이 편하다. 여기서 이러한 곱셈공식과 활용법을 연구하자.

곱셈공식 1

$(a+b)^2 = a^2 + 2ab + b^2$

(합의 제곱)

곱셈공식 2

$(a-b)^2 = a^2 - 2ab + b^2$

(차의 제곱)

풀이 〔1〕 $(a+b)^2$

$= (a+b)(a+b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

〔2〕

$$\begin{array}{rr} & a + b \\ \times) & a + b \\ \hline & a^2 + ab \\ & ab + b^2 \\ \hline & a^2 + 2ab + b^2 \end{array}$$

풀이 〔1〕 $(a-b)^2$

$= (a-b)(a-b)$

$= a^2 - ab - ab + b^2$

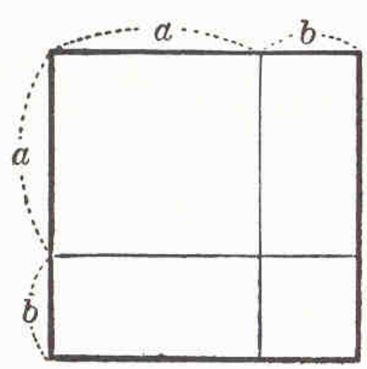
$= a^2 - 2ab + b^2$

〔2〕

$$\begin{array}{rr} & a - b \\ \times) & a - b \\ \hline & a^2 - ab \\ & -ab + b^2 \\ \hline & a^2 - 2ab + b^2 \end{array}$$

〔주의〕 $(a+b)^2$을 a^2+b^2, $(a-b)^2$을 a^2-b^2으로 하지 않도록 주의를 해야 한다.

물음 1. 오른편 그림을 보고 면적 관계를 이용하여, 합의 제곱공식이 성립함을 증명하여라.



BINOMIAL EXPANSIONS

$(x + y)^0 = \mathbf{1}$

$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + \mathbf{2}xy + y^2$

$(x + y)^3 = x^3 + \mathbf{3}x^2y + \mathbf{3}xy^2 + y^3$

$(x + y)^4 = x^4 + \mathbf{4}x^3y + \mathbf{6}x^2y^2 + \mathbf{4}xy^3 + y^4$

$(x + y)^5 = x^5 + \mathbf{5}x^4y + \mathbf{10}x^3y^2 + \mathbf{10}x^2y^3 + \mathbf{5}xy^4 + y^5$

$(x + y)^6 = x^6 + \mathbf{6}x^5y + \mathbf{15}x^4y^2 + \mathbf{20}x^3y^3 + \mathbf{15}x^2y^4 + \mathbf{6}xy^5 + y^6$

Pascal's Triangle

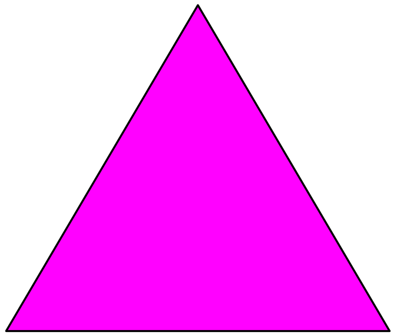
						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1

The Connection Between Pascal's Triangle and the Fibonacci Sequence

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

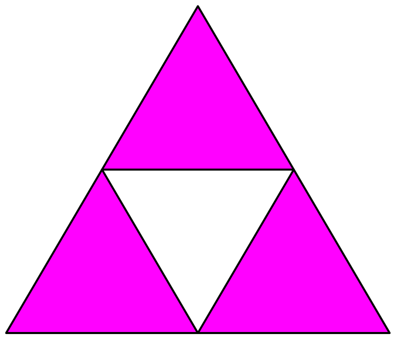
Sierpinski Triangle



Stage 0

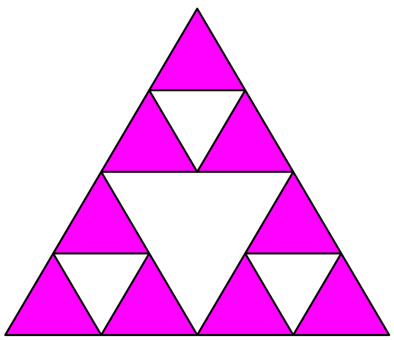
If the area of the shaded region in Stage 0 is 1, find the area of the shaded region in the other stages.

Area = 1



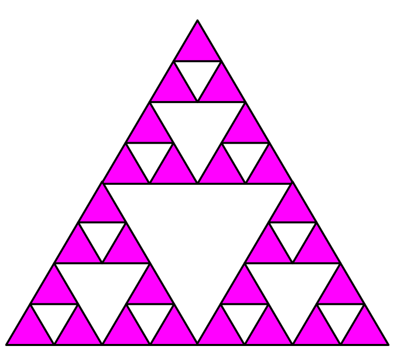
Stage 1

Area =



Stage 2

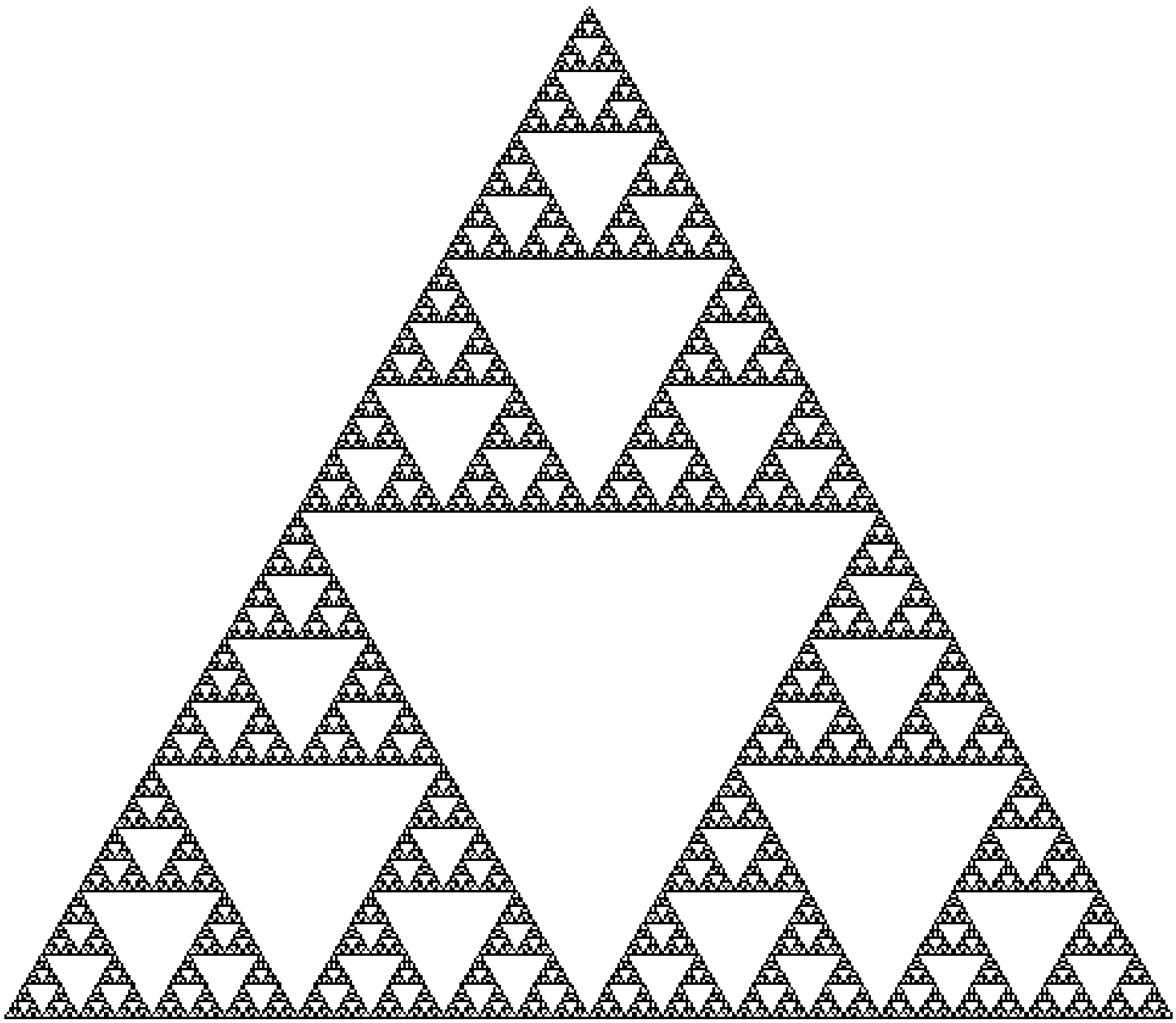
Area =



Stage 3

Area =

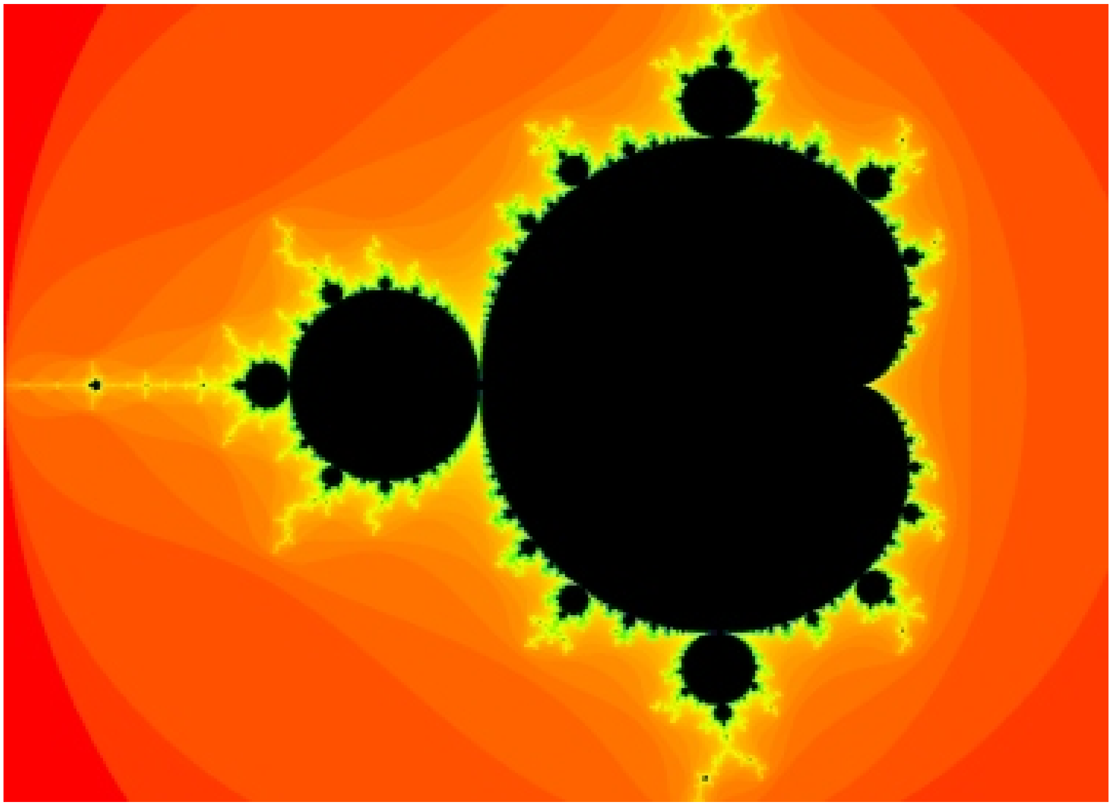
Sierpinski Triangle



Sierpinski Pyramid

Postage stamp issued by Hungary in 1996 on the occasion of a Mathematical Congress held in Hungary.

Mandelbrot Set



Fractal Landscape

Merriam-Webster's COLLEGIATE® DICTIONARY

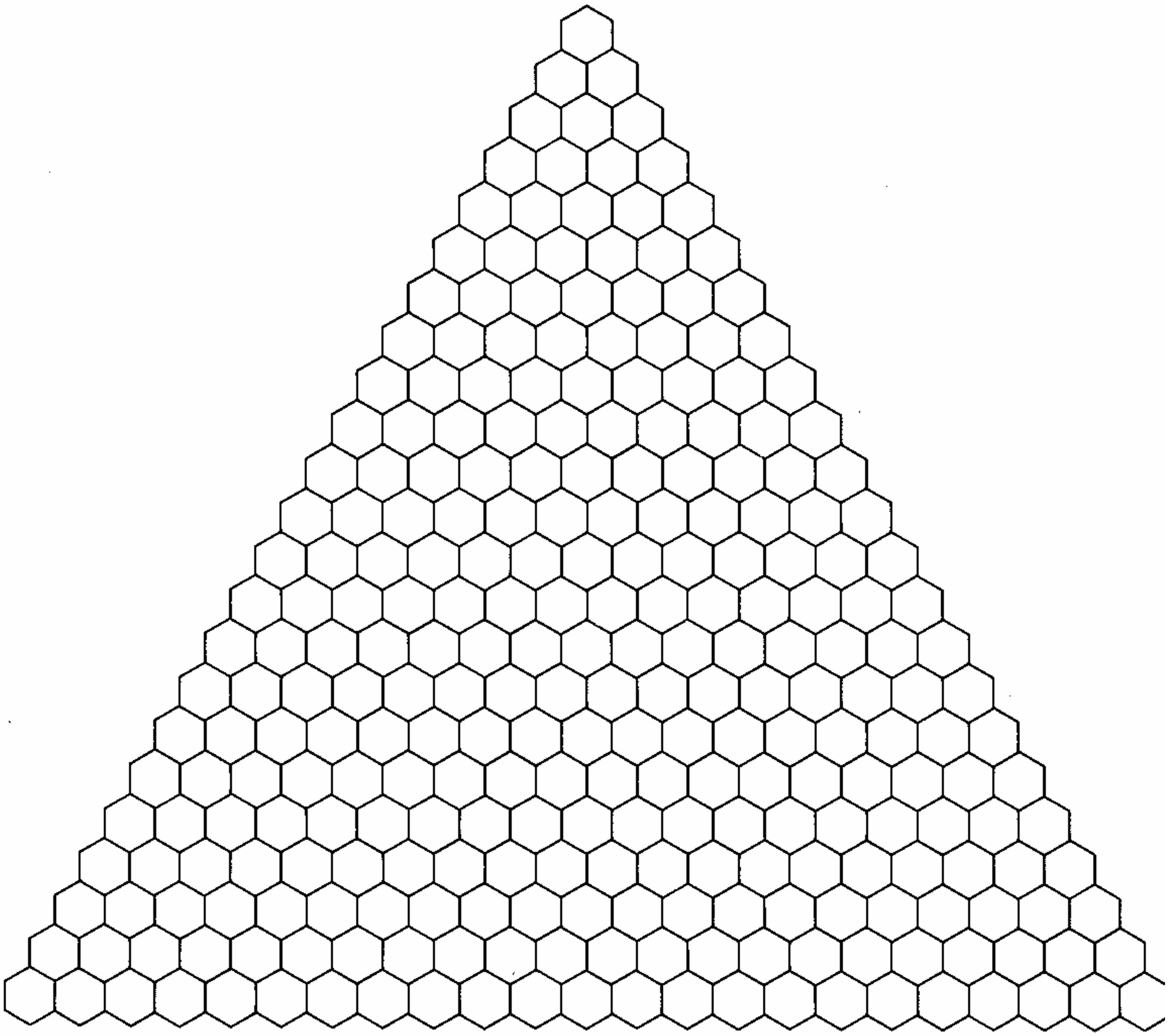
Atlas | Reverse Dictionary | Rhyming Dictionary

Collegiate® Dictionary | Collegiate® Thesaurus | Unabridged

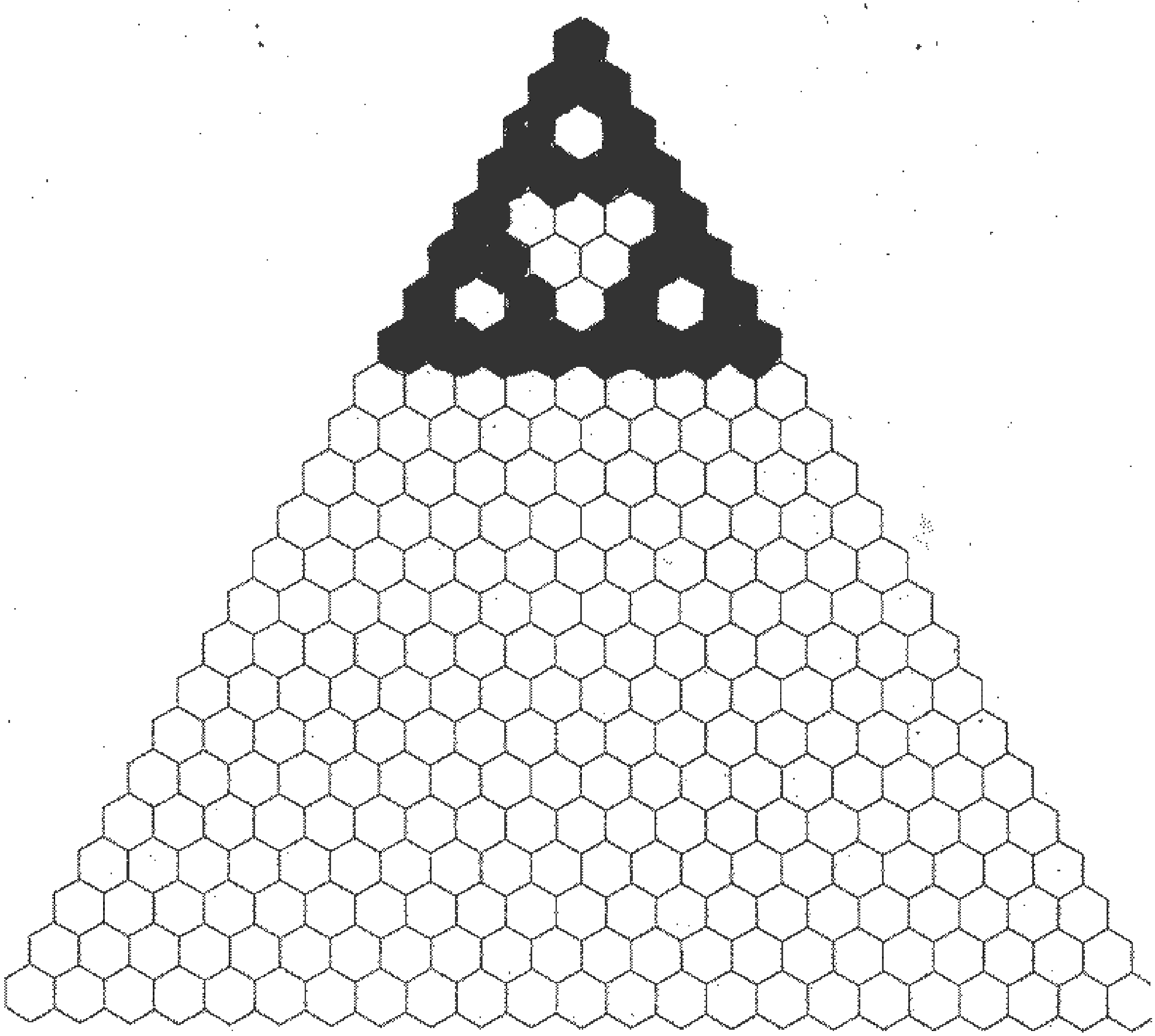
One entry found for **fractal**.

Main Entry: **frac·tal**
Pronunciation: 'frak-t&l
Function: *noun*
Etymology: French *fractale,* from Latin *fract*us broken, uneven (past participle of *frangere* to break) + French -*ale* -al (noun suffix)
Date: 1975
: any of various extremely irregular curves or shapes for which any suitably chosen part is similar in shape to a given larger or smaller part when magnified or reduced to the same size
- fractal *adjective*

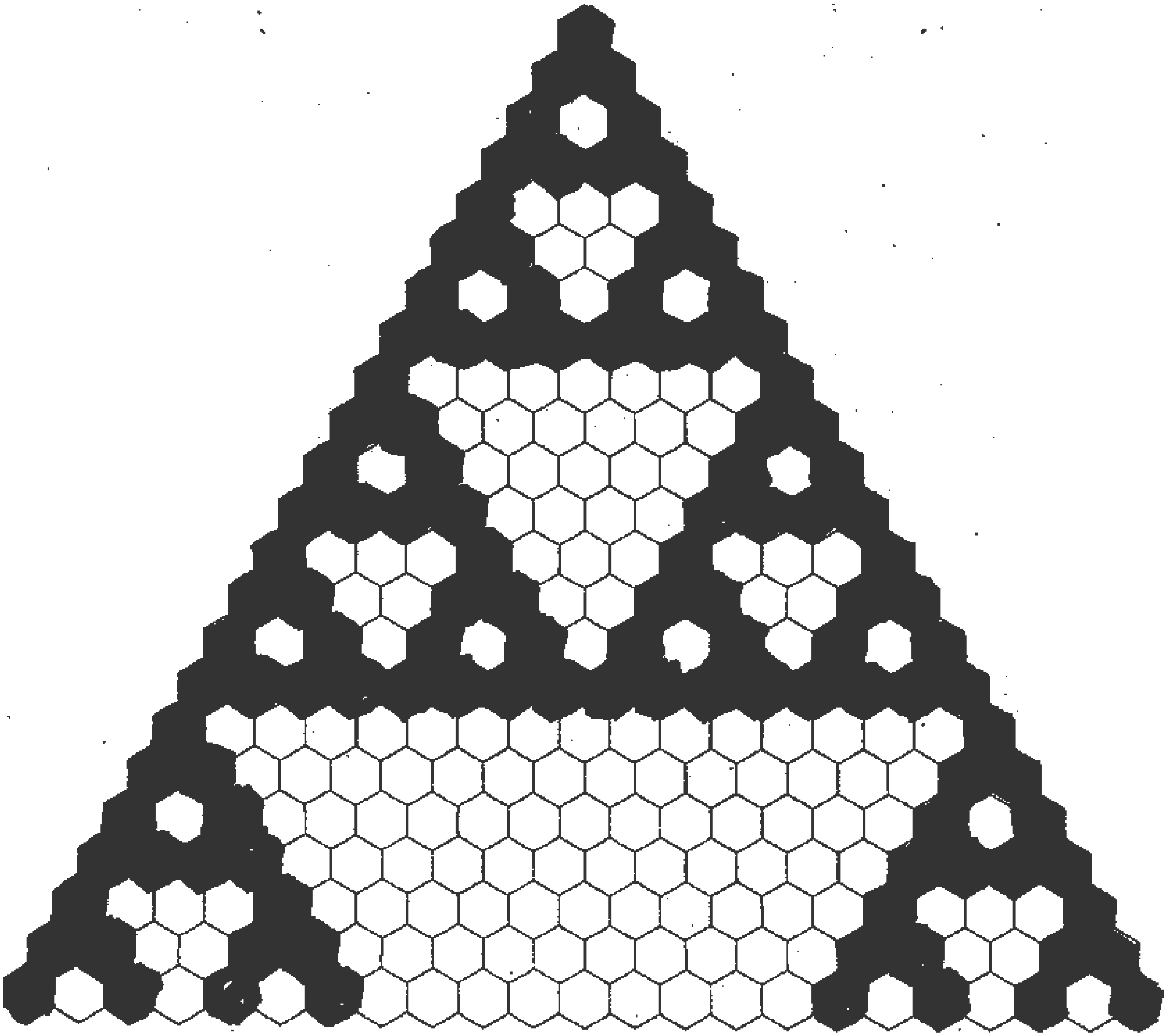
Group Project: Fill in using numbers from Pascal's Triangle, then color in each polygon that contains an odd number.

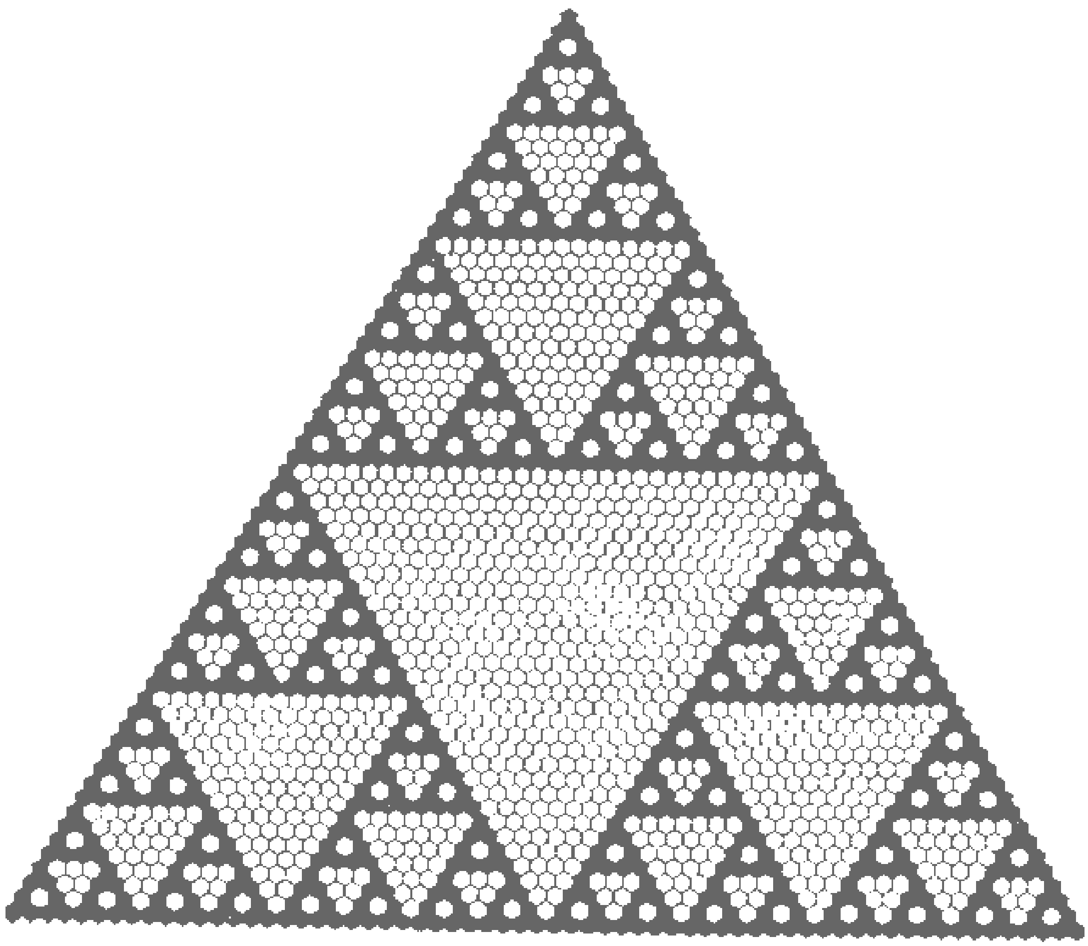


Pascal's Triangle



Pascal's Triangle





Iterated Functions

Definition: If x is in the domain of a function f, then the sequence below is called an *orbit* of x for f. Each member of the sequence is called an *iterate* of x for f.

$$x, \quad f(x), \quad f(f(x)), \quad f(f(f(x))), \ldots$$

A Regular Function

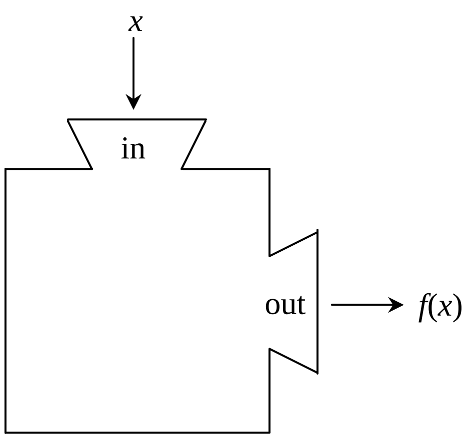


Figure 1

An Iterated Function

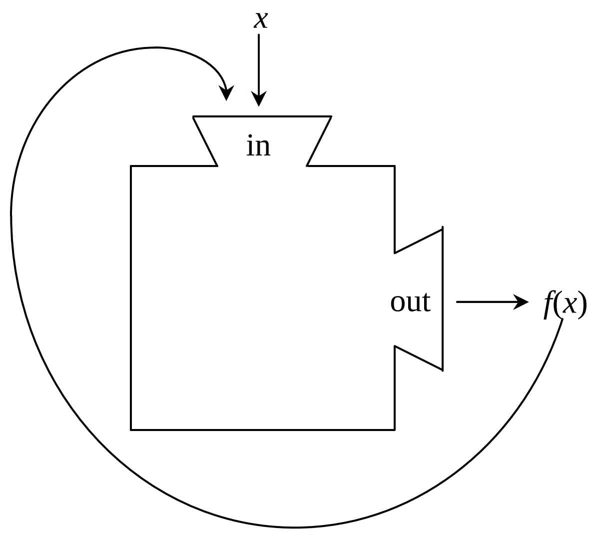


Figure 2

EXAMPLE Write the orbit of 0 for $f(x) = \sqrt{1+x^2}$.

First iterate: $x = 0$

Second iterate: $f(0) =$

Third iterate: $f(f(0)) =$

Fourth iterate: $f(f(f(0))) =$

Fifth iterate: $f(f(f(f(0)))) =$

EXAMPLE Write the orbit of 2 for $f(x) = 1 + \frac{1}{x}$.

First iterate: $x = 2$

Second iterate: $f(2) =$

Third iterate: $f(f(2)) =$

Fourth iterate: $f(f(f(2))) =$

Fifth iterate: $f(f(f(f(2)))) =$

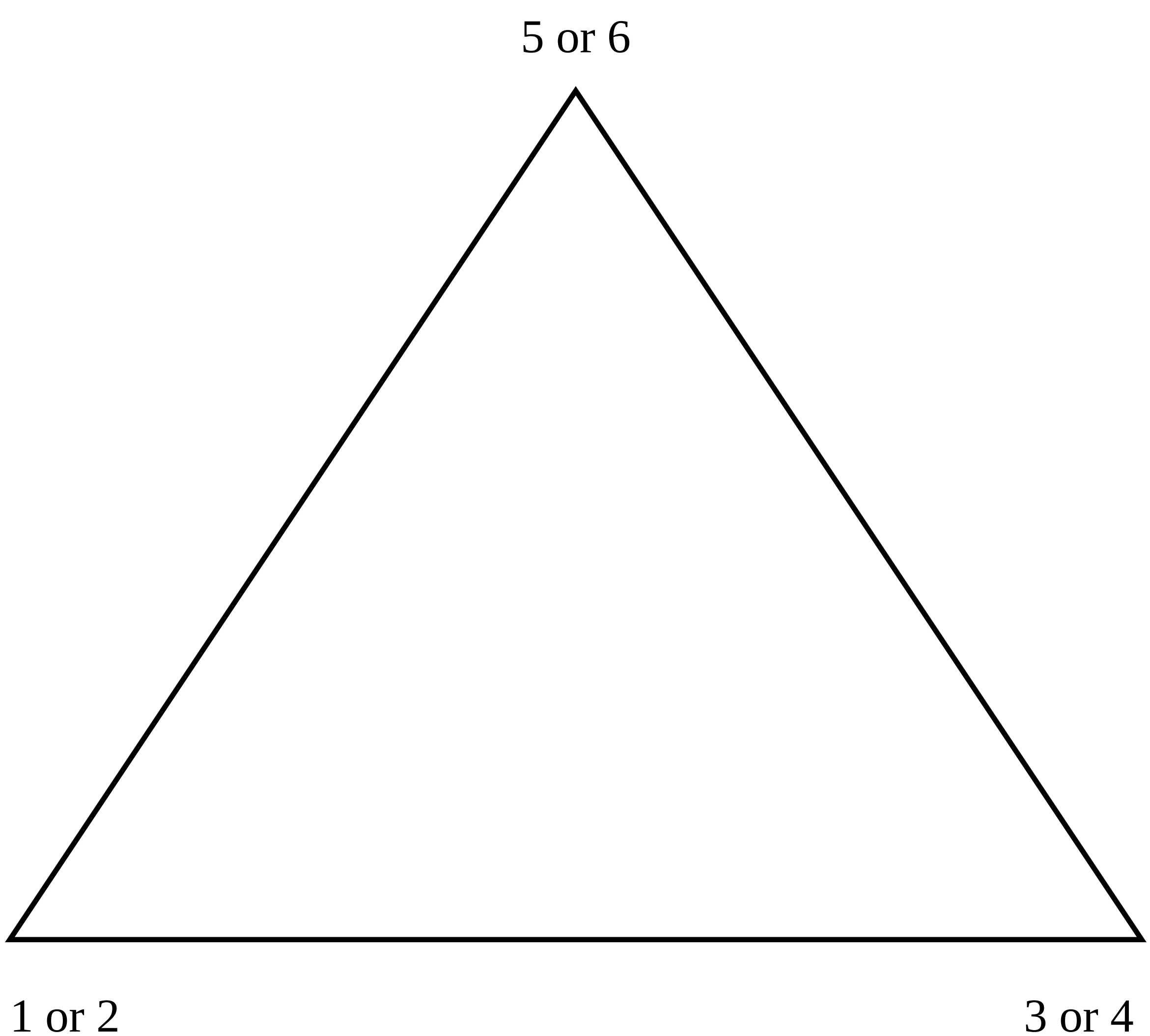
The Chaos Game - Lines

Step 1. Pick any point within the triangle as the starting point.

Step 2. Roll a die. The number that appears will indicate which vertex you are to move toward.

Step 3. Draw a line segment from your starting point to the point that is halfway to the indicated vertex.

Step 4. The point halfway to the vertex is your new starting point. Go to Step 2 and repeat the process.



The Chaos Game - Dots

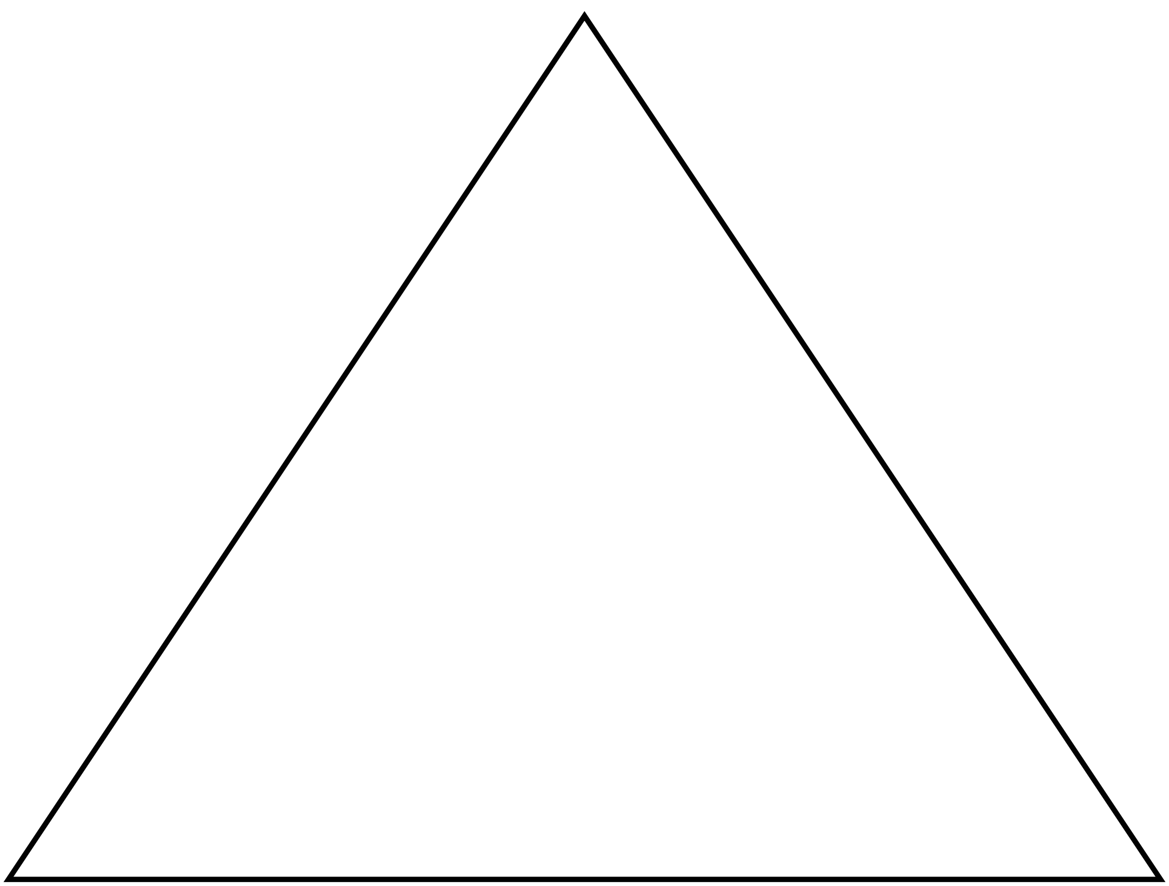
Step 1. Pick any point within the triangle as the starting point.

Step 2. Roll a die. The number that appears will indicate which vertex you are to move toward.

Step 3. Place a dot at the point that is halfway to the indicated vertex.

Step 4. The point halfway to the vertex is your new starting point. Go to Step 2 and repeat the process.

5 or 6



1 or 2

3 or 4